The flow over a sphere of radius a is affected by the entire front part of the sphere. Therefore in calculating the time taken by the fluid to move along an arbitrary streamline in the layer δ it is necessary to integrate within the limits θ, $\pi-\theta$ (θ is the angle between the radius vector and the polar axis, which coincides with the direction of motion of the unperturbed flow; the coordinate origin is located at the center of the sphere).

The diffusion time averaged over the entire surface of the sphere

$$\tau = \frac{2}{4\pi a^2}\int_0^{\frac{\pi}{2}} 2\pi a \sin\theta \int_\theta^{\pi-\theta} \frac{d(a\psi)}{\frac{1}{\delta}\int_0^\delta v_\theta(y,\psi)\,dy}\, d(a\theta). \quad (8)$$

We will estimate the diffusion flow to a spherical droplet moving at Re ≪ 1. In accordance with the Hadamard-Rybchinskii solution, the tangential velocity

$$v_\theta = u\varphi\sin\theta, \quad (9)$$

where u is the velocity of the droplet;

$$\varphi = \frac{1}{2(1+\mu)} + \frac{1+3\mu}{2(1+\mu)}\frac{y}{a}.$$

From Eqs. (7)–(9) we obtain an equation for estimating the thickness of the diffusion layer:

$$\delta^2 = 4\beta^2\frac{Da}{u}\,\frac{1+\mu}{1+(1+3\mu)\frac{\delta}{2a}}\int_0^{\frac{\pi}{2}}\sin\theta\int_\theta^{\pi-\theta}\frac{d\psi}{\sin\psi}\,d\theta. \quad (10)$$

Integrating and keeping in mind that $Nu = a/\delta$, $Pe = ua/D$, we reduce Eq. (10) to the form

$$Pe = (16\ln 2)\,\beta^2(1+\mu)\frac{Nu^3}{1+3\mu+2Nu}. \quad (11)$$

For a bubble ($\mu \ll 1$, $Pe \gg 1$) expression (11) takes the form

$$Nu = \frac{0.42}{\beta}Pe^{\frac{1}{2}}. \quad (12)$$

If $\mu \to \infty$ (solid particle) and $Pe \gg 1$, Eq. (11) becomes

$$Nu = \frac{0.65}{\beta^{2/3}}Pe^{\frac{1}{3}}. \quad (13)$$

When the viscosities are comparable ($\mu \approx 1$), we have

$$Nu = \frac{0.42}{\beta}Pe^{\frac{1}{2}}(1+\mu)^{\frac{1}{2}}. \quad (14)$$

The numerical coefficients in (12)–(14) coincide with the exact values [1] if β is set equal to 0.91 in (12) and (14) and 1.02 in (13). This confirms the validity of the above estimate of β and indicates that this coefficient depends only very slightly on viscosity.

It may be assumed that the function $Nu(\mu)$, determined from Eq. (11), does not have singularities in the interval of variation of viscosity $0 \le \mu \le \infty$. Therefore the quantity β in Eq. (11) may be taken equal to its mean value of 0.97. When the above value of the coefficient β is employed, the diffusion flux is calculated correct to approximately ±6%.

Since we have assumed the stationarity of the concentration field, the method proposed is applicable for times t much greater than the time τ during which as a result of diffusion to the surface of the body the particles are displaced through a distance equal to the thickness of the diffusion layer δ, i.e.,

$$t \gg \tau = \frac{\delta^2}{2\beta^2 D}.$$

In the stationary regime the diffusion time is equal to the convection time. The latter is equal in order of magnitude to a/u, which makes it possible to put the stationarity condition in the form $t \gg a/u$, a form convenient for practical calculations.

NOTATION

a is the radius of droplet (sphere); C is the concentration, C_0 is the same remote from the surface; D is the diffusion coefficient; j is the diffusion flux; $n = C/C_0$ is the dimensional concentration; t is time; U and u are the velocity of liquid and droplet, respectively; $v_\theta v_x$ is the tangential component of velocity; v_y is the normal component of velocity; x is the coordinate along surface of body; y is the coordinate along normal to surface; $Nu = a/\delta$ is the Nusselt number; $Pe = ua/D$ is the Peclet number; Re is the Reynolds number; $\beta = 1/(2\alpha)^{1/2}$ is a coefficient; δ is the thickness of diffusion layer; $\zeta = y/\lambda$ is a dimensionless coordinate; ψ, θ are polar angles; λ is the scale in the y direction; μ_1, μ_2 is the dynamic viscosity of liquid inside and outside droplet, $\mu = \mu_1/\mu_2$; $\xi = t/\tau$ is dimensionless time; τ is the time scale.

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CALCULATION OF THE TEMPERATURE FIELD IN THE WALL OF A SEGMENTED-ELECTRODE MHD CHANNEL ON AN EHDA-9/60 INTEGRATOR

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It is now becoming clear that the use of ceramic materials in the channels of MHD generators operating on combustion products at 2500–3000° K must be extremely limited. The experimentally observed damage and the interaction of the plasma with the duct walls exclude the possibility of using uncooled systems over extended periods, but it is to be expected that water-cooled metal walls will be sufficiently stable if properly designed as box sections with thin intervening layers of insulation.

As shown in [1], the use of thin insulators (0.1 of electrode pitch) is justified from the power engineering standpoint and at the same time makes it possible to keep the insulator temperature sufficiently low—not more than 1200° C, a temperature at which the channel structure may be expected to be sufficiently stable.

A cross section through a cooled box-electrode and insulator is shown in Fig. 1. In order to obtain the temperature distribution in the electrode metal and the insulator we simulated the stationary temperature field on conductive paper using an EHDA-9/60 integrator. In so doing we employed the analogy between the flow of current in a conductive medium and the process of heat propagation in the wall regions of an MHD duct with variable conductivity.

The electrical conductivity of the paper σ, the analog of the thermal conductivity λ of the simulated medium, varies discontinuously on passing from one part to another, but remains constant within each part.

The boundary conditions are determined on the assumption that the problem is periodic and that the centers of electrode and insulator are axes of symmetry. On them $dT/dx = 0$.

At the outer edge of the thermal layer the gas temperature is given, $T_g = \text{const}$; similarly, on the coolant side $T_c = \text{const}$.

The thermal resistance of the thermal layer at the wall was simulated with an additional layer of paper whose thickness is given by

$$\delta = \lambda/\alpha. \quad (1)$$

Here, λ is the thermal conductivity of the layer of gas or liquid, and α is the heat transfer coefficient, assumed constant over the entire surface of contact between the plasma and the electrode and insulator and also over the entire electrode cooling surface.

Calculations [2] show that in the possible range of surface temperature variation (600–1200° K) α_g varies from 0.71 to 0.56 kW/$/\text{m}^2 \cdot$deg. This can be taken into account in the second approximation by correspondingly varying the zone width b (Fig. 1).

The assumption that α is constant simplifies the problem and makes it possible to draw the necessary qualitative conclusions.

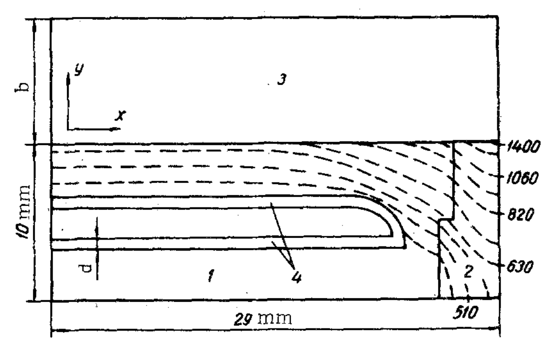

Fig. 1. Diagram of model (at outer edge of thermal layer $\varphi = 1.0 = \text{const}$, $T_2 = 2800^\circ$K; on the coolant side $\varphi = 0 = \text{const}$, $T_c = 323^\circ$ K): 1) metal electrode, 2) insulator, 3) layer of gas, 4) layer of liquid.

The error introduced by the condition $\alpha_c = \text{const}$ lies within the limits of accuracy of the construction of the model.

Using the physical similarity condition $\sigma = n\lambda$ and expression (1), we write the scale relation in the form

$$\sigma_i/\lambda_i = \sigma_3/\alpha_g b = \sigma_4/\alpha_c d. \quad (2)$$

Calculations [2] give: $T_g = 2800^\circ$ K, $T_{gw} = 600^\circ$ K, $G_g = 22$ kg/ /sec, $G_c = 0.84$ l/sec, $\alpha_g = 0.71$ kW/m$^2 \cdot$deg. This corresponds to the most heavily loaded inlet section of the duct.

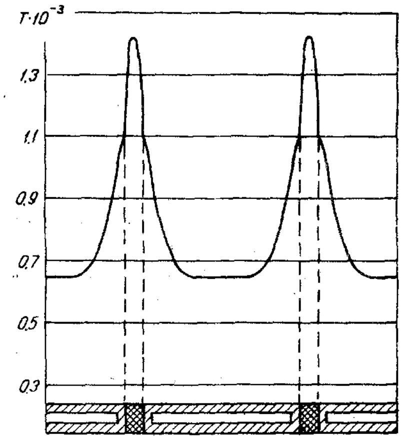

Fig. 2. Distribution of temperature T, °K, at surface of electrode and insulator.

It was assumed that $\lambda_{el} = 23.3$ W/m$\cdot$deg, $\lambda_{insul} = 1.65$ W/m$\cdot$ $\cdot$deg. A model of the channel wall was formed of conductive paper of different conductivity in accordance with condition (2).

The temperature distribution obtained (Fig. 2) indicates that the corners of the cooled electrodes are zones of high temperature (up to 1100° K) relative to the center of the electrode.

This should be taken into account in examining possible mechanisms of current flow through cooled electrodes and the relationship between current and electrode temperature.

The above calculations did not allow for the fact that in the Hall-effect generator considered there is a nonuniform current density distribution near the electrodes and corresponding Joule losses (the term j^2/σ in the energy equation) in the boundary layer.

Clearly, the temperature at the corners of the electrode will be even higher, since the current density in that zone is especially high.

Disregarding this secondary effect, we may conclude that the temperature of the thin insulator lies within permissible limits (up to 1400° K at the surface), so that there is a good chance of its giving extended service in the channel.

NOTATION

j is the electric current density; σ is the electrical conductivity of the paper; T is the absolute temperature; λ is the thermal conductivity of the medium; δ is the thickness of the simulated layer; n is the scale factor; α_g is the coefficient of heat transfer from the gas to the wall; α_c is the coefficient of heat transfer from the wall to the coolant; T_g is the stagnation temperature of the gas flow; T_{gw} is the gas temperature at the wall; G_g is the flow rate of the combustion products; G_c is the flow rate of the coolant.

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